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*Technical Memorandum 33-609*

*Orbit Determination for Low-Thrust Spacecraft:  
Concepts and Analysis*

*James P. McDanell*

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**JET PROPULSION LABORATORY  
CALIFORNIA INSTITUTE OF TECHNOLOGY  
PASADENA, CALIFORNIA**

April 15, 1973

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## PREFACE

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# ORBIT DETERMINATION FOR LOW-THRUST SPACECRAFT: CONCEPTS AND ANALYSIS\*

## Abstract

This paper re-evaluates Earth-based orbit determination capability for SEP spacecraft in the light of recent developments both in multi-station tracking concepts and in thrust subsystem error modeling. Five different tracking strategies are applied to a 15 day segment of an Encke rendezvous mission. Both optimal and suboptimal orbit determination performance are determined for a wide range of process noise parameter values. The multi-station tracking techniques are found to be extremely effective, reducing orbit determination errors by orders of magnitude over that obtained with conventional single-station tracking. Explicitly differenced multi-station data (QVLBI) is found to be least sensitive to gross modeling errors, but if a reasonably good process noise model is available, explicit differencing is not required.

## I. Introduction

Solar electric propulsion (SEP) is characterized by high level stochastic nongravitational accelerations resulting from random variations in the thrust process. The random accelerations are roughly three orders of magnitude larger than those due to gas leaks, solar pressure variations, etc. on ballistic missions and at such levels constitute the dominant error source for Earth-based interplanetary navigation of SEP spacecraft. (1)

Since reduction of thrust subsystem errors to ballistic levels does not appear to be feasible, the successful navigation of SEP missions depends upon making the orbit determination process more tolerant of stochastic forces. As a first step toward this end it is clear that some form of stochastic model is required to represent the random thrust process, and this implies the need for a sequential filter (2) to accommodate the resulting process noise model. Unfortunately, even with optimal sequential filtering under the optimistic assumption that the process noise model is a perfect representation of the physical random process, orbit determination capability for SEP spacecraft remains grossly inferior to that for ballistic craft when conventional Earth-based tracking is assumed.

Adaptive filtering techniques (3,4) may be useful for SEP missions as a means of identifying and updating the process noise model in real time. But it is clear that such techniques can only enable the filter to approach more closely in operation the theoretical optimal performance indicated when the process noise model is assumed correct and therefore needs no updating. If the idealized (optimal) filter performance is unsatisfactory, as is generally the case for SEP, no filter algorithm,

however elegant, can remedy the situation. Something more fundamental is needed; i. e., the information content of the data available to the filter must be upgraded.

One superficially attractive approach would be to reduce the uncertainties at their source, i. e., to measure the actual nongravitational accelerations in real time with precision onboard accelerometers and make this data available to the orbit determination filter. Unfortunately, the instrumentation requirements (resolution and alignment) for this approach to be effective are prohibitive. Specifically, to approach ballistic orbit determination accuracy the acceleration uncertainties must be reduced about three orders of magnitude. Since the a priori uncertainties are of the order of one percent of the nominal thrust, this means that the instrumentation must be capable of measuring the low thrust accelerations (which are of the order of  $10^{-5}$  g) to an accuracy of 0.001 percent. The alignment of the accelerometer axes in inertial space must be known to a comparable accuracy, or about  $10^{-5}$  radian. These requirements appear to be well beyond the state of the art. (5,6) Furthermore, there is the question of reliability. In each of two flight tests of a low-g electrostatic accelerometer, the instrumentation failed after a brief period of operation. (5,6)

Fortunately, there is another approach that is much more promising, namely, the development of an Earth-based data type that is relatively insensitive to stochastic acceleration effects. Ondrasik and Rourke (7,8,9) have shown that differencing simultaneous range-rate data from widely separated stations produces a data type that contains the essential right ascension and declination information and is largely unaffected by stochastic accelerations. This technique is called "two-station tracking" or QVLBI (Quasi-Very Long Baseline Interferometry). Although still in its infancy and not yet proven in operations, this simple concept promises to be the breakthrough needed for low thrust navigation to become competitive with ballistic.

With the emergence of QVLBI as a viable tracking strategy a re-evaluation of low thrust orbit determination capability is needed, taking into account recent developments in SEP thrust subsystem error modeling. There are several important questions to be answered:

- 1) How effective is QVLBI compared with conventional tracking for SEP missions?
- 2) Is explicit differencing of the simultaneous data necessary to achieve the desired effect?

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3) What degradation of performance occurs as the result of an incorrect thrust subsystem error model?

4) What are guidelines for conservative error modeling; i. e., are some types of model errors more damaging than others?

5) What is the relative importance of the various error sources?

This paper provides preliminary answers to these and related questions, using a 1980 rendezvous with the Comet Encke as a representative SEP mission. First, the basic thrust subsystem stochastic error model is described and baseline values for the error model parameters are given. Five different tracking strategies, including QVLBI, are defined and applied to a 15 day segment of the 950 day Encke mission, using the baseline values of thrust subsystem error model parameters to determine the baseline orbit determination performance. Then, for three of the five tracking strategies, the error model parameters are varied. Both optimal and suboptimal performance are determined for a wide range of parameter values. The total orbit determination errors for QVLBI and single-station tracking are also broken down and analyzed according to individual error sources.

## II. Thrust Subsystem Error Model

The results of any SEP orbit determination study depend very strongly upon the assumptions made about the stochastic nongravitational accelerations. For this reason, considerable effort has been directed by the SEP Navigation Development Team at JPL toward identifying the major thrust subsystem error sources and developing a satisfactory model for their representation. The resulting "process noise model" is the basis for the present study. The basic characteristics of the error model will be outlined in this section.

The stochastic accelerations are assumed to be due solely to variations in the thrust vector, which is represented by its magnitude and two orientation angles (clock and cone angles with respect to the sun-probe vector). The errors in each orientation angle and in thrust magnitude are assumed to be independent and to consist of a zero-mean time-varying random component superimposed upon a bias. Each time-varying component is modeled as a first order Gauss-Markov random process; <sup>(10)</sup> i. e., each component is assumed to satisfy a stochastic differential equation of the form

$$\dot{\xi} = -(1/\tau)\xi + w \quad (1)$$

where  $w$  is a Gaussian white noise process such that

$$E[w(t)] = 0 \quad (2)$$

$$E[w(t_1)w(t_2)] = (2\sigma^2/\tau) \delta(t_1 - t_2)$$

Each random process of this form is completely characterized by two parameters: the standard deviation ( $\sigma$ ), which is a measure of uncertainty, and the correlation time ( $\tau$ ), which is a measure of transience. The following system of notation will be used:  $\sigma_{Tb}$ ,  $\sigma_T$ , and  $\tau_T$  denote the standard

deviation of the bias, standard deviation of the time-varying component, and correlation time, respectively, for the thrust magnitude error;  $\sigma_{\phi b}$ ,  $\sigma_{\phi}$ ,  $\tau_{\phi}$  are the corresponding parameters for the thrust orientation angles. The two angles are assumed to have identical statistical properties so that a single set of parameters applies to both.

To obtain baseline values for the standard deviations of the process noise model it was necessary to identify the contributing error sources, determine the expected error in each parameter, and compute the corresponding effect on the thrust process. For the orientation angles the error analysis is relatively simple. The bias in the angles is due mainly to grid warpage and is typically about 2 degrees (.035 radian). The time-varying angle components represent the random pointing error of the spacecraft within the deadband of the attitude control sensors. This is expected to be less than one degree, or approximately .01 radian.

Analysis of thrust magnitude errors is somewhat more complicated, as there are a number of thrust subsystem parameters contributing to the overall magnitude error. The nominal thrust magnitude is given by

$$T = K \left( \frac{\eta_1 + \sqrt{2} \eta_2}{\eta_1 + 2 \eta_2} \right) I_B \sqrt{V_B} \frac{1}{\cos \theta} \zeta \quad (3)$$

where

$I_B$	= beam current
$V_B$	= net accelerating potential
$\frac{1}{\cos \theta}$	= beam divergence factor
$\eta_1$	= singly ionized fraction of mass flow
$\eta_2$	= doubly ionized fraction of mass flow
$\zeta$	= charge exchange parameter
$K$	= constant

A linear expansion of Eq. (3) about nominal parameter values gives the following expression for the relative thrust magnitude error ( $\Delta T/T$ ).

$$\frac{\Delta T}{T} = \frac{\Delta I_B}{I_B} + \frac{1}{2} \frac{\Delta V_B}{V_B} + (2 - \sqrt{2}) \frac{\eta_2}{\eta_1} \left[ \frac{\Delta \eta_1}{\eta_1} + \frac{\Delta \eta_2}{\eta_2} \right] + \frac{\frac{\Delta \cos \theta}{\cos \theta} + \frac{\Delta \zeta}{\zeta}}{\cos \theta} \quad (4)$$

From Eq. (4) and from available information about the range of parameter variations, the contribution of each parameter to the overall thrust magnitude error (as a percent of nominal thrust) can be determined. This information is summarized in Table 1. The dominant error sources are statistically independent, so the total thrust error is the root-sum-square (RSS) of the individual errors.

The standard deviations in Table 1 are for a single thruster. When more than one thruster is operating, the standard deviations for the process

Table 1 Thrust Subsystem Error Summary

Parameter	Nominal Value	Calibration Accuracy (%)	Variation with Time (%)	Contribution to Thrust Error (% of Nominal Thrust)	
				$\sigma_{Tb}$	$\sigma_T$
$I_B$	Programmed	$\pm 0.5$	$\pm 1.5$	0.5	1.5
$V_B$	1500 V	$\pm 0.5$	$\pm 1.0$	0.25	0.5
$\cos \theta$	0.96	$\pm 2.0$	$\pm 3.0$	2.0	3.0
$\eta_1$	0.80 - 0.85	$\pm 1.0$	$\pm 5.0$	0.02 - 0.05	0.10 - 0.25
$\eta_2$	0.04 - 0.07	$\pm 20.0$	$\pm 25.0$	0.5 - 1.0	0.50 - 1.25
$\zeta$	0.995	$\pm 0.15$	$\pm 0.15$	0.15	0.15
Total (RSS)				2.2	3.5

noise model are as given in Table 2. The total error is the RSS of individual thruster errors in each case with the exception of the pointing error from the attitude control system ( $\sigma_\phi$ ), which is independent of the number of thrusters.

Table 2 Process Noise Model Standard Deviations

Number of Operating Thrusters	Thrust Magnitude (% of Nominal Thrust)		Orientation Angles (radians)	
	$\sigma_T$	$\sigma_{Tb}$	$\sigma_\phi$	$\sigma_{\phi b}$
1	3.5	2.2	0.010	0.035
2	2.5	1.6	0.010	0.025
3	2.0	1.3	0.010	0.020
4	1.75	1.1	0.010	0.0175
5	1.6	1.0	0.010	0.016

The present study is based on a 15 day segment of the 950 day Encke rendezvous mission terminating 5 days before encounter. During this time power matching requirements dictate that there will be four thrusters operating. Thus, the standard deviations from Table 2 corresponding to four thrusters were used as baseline values. The indicated thrust magnitude error of 1.75 percent represents acceleration errors of  $.6$  to  $.8 \times 10^{-8}$  km/sec<sup>2</sup> during this particular mission segment.

The correlation times required for the error model are somewhat more nebulous than the standard deviations. Observed variations in the quantities contributing to the thrust magnitude error indicate relatively long correlation times, of the order of days to weeks. Since the longer correlation times in this range approach a bias effect, of which the orbit determination process is relatively tolerant, in the interest of conservative modeling a value for  $\tau_T$  toward the lower end of this spectrum, namely, 5 days, was selected as the baseline value. Pointing errors are higher in frequency and more transient than the thrust magnitude fluctuations. Indeed, correlation times of the order of hours appear to be appropriate. Accordingly, a baseline value of 3 hours was selected for  $\tau_\phi$ .

Conspicuously absent from this error model is any mention of mass variations, which, if present, of course, also contribute to the stochastic acceleration. The fundamental thrust subsystem parameters do not affect mass directly, but three of them, namely,  $I_B$ ,  $\eta_1$  and  $\eta_2$ , ultimately affect mass through variations in the mass flow rate  $\dot{m}$ . Indeed, a linear expansion for  $\dot{m}$  analogous to Eq. (4) indicates that the time-varying error component in  $\dot{m}$  may be as large as  $\pm 5\%$ . This seemingly large

error is actually negligible for the following reasons: First, the variation in fuel (i.e., mass) expended over the time period of a typical tracking interval is only a small portion of the total fuel for the 950 day mission, which in turn is a relatively small portion (about 30%) of the total spacecraft mass; e.g., for the particular 15 day period of interest in this study, a 5% variation in mass flow rate represents a maximum of .09% variation in the total mass, which is clearly negligible with respect to the thrust magnitude errors. Furthermore, since the mass variation is the integral of the error in  $\dot{m}$ , higher frequency components are attenuated, and the slowly varying components, even if they were not of negligible magnitude, could be adequately represented as a bias over a short tracking interval. The foregoing observations do not preclude the possibility of a large initial uncertainty in mass due to the cumulative effect of mass flow rate errors over a long period of time preceding the data arc. In this study an initial uncertainty of 100 kg (out of a total spacecraft mass of 1200 kg) was assumed.

### III. Filter Configuration and Tracking Strategies

The orbit determination filter used for this study was a discrete form of the Kalman sequential filter, (2, 10) with the additional capability of "considering" the effect of specified parameters without explicitly estimating them. A companion evaluation algorithm allowed separate specification of the process noise parameters for the filter model and the assumed real-world environment. The stochastic thrust errors were treated as piecewise constant functions over one hour time intervals, closely approximating the continuous Gauss-Markov process of Eq. (1) in the manner suggested by Russell and Curkendall. (11) The details of the filter algorithm are given in the Appendix.

The estimated parameters, in addition to the spacecraft state, were the initial mass, the biases and stochastic components of the two thrust vector orientation angles, and the stochastic component of the relative thrust magnitude  $\Delta T/T$ . The bias in relative thrust magnitude was not included because, under the assumption that the nominal mass is constant during the tracking interval (it actually decreases about 2% in this case), the effect of a bias in relative thrust magnitude is indistinguishable from that of a bias in the mass (i.e., they are perfectly correlated). Therefore, it is redundant for the filter to explicitly estimate both quantities. The large initial mass uncertainty (100 kg) assumed for this study effectively absorbs the bias component in thrust magnitude, allowing the latter to be neglected. A priori uncertainties for all the above-mentioned quantities are given in Table 3, along with the assumed data accuracies. The DSN station locations were considered as an error source, but not explicitly estimated by the filter.

Two single-station (SS) and three multi-station (MS) tracking strategies were investigated in this study. For ease of reference in the sequel, the following abbreviated terminology will be used to refer to specific combinations of data types.

- 1) SS Doppler-only: two-way range-rate data from a single station.
- 2) SS conventional: two-way range and range-rate from a single station.



Table 3 Key Assumptions for the Baseline Case

Description	Numerical Value
Initial S/C State Uncertainty	
RSS Position	17,300 km
RSS Velocity	17.3 m/s
Mass	100 kg
Thrust Magnitude Errors	
Bias Component	(absorbed in mass)
Time-Varying Component	1.75%
Correlation Time	5 days
Orientation Angle Errors	
Bias Component	0.0175 rad
Time-Varying Component	0.010 rad
Correlation Time	3 hr
DSN Station Location Errors	
Spin Radius	1.5 m
Longitude	3.0 m
Parallel to Spin Axis	5.0 m
Data Noise	
Range Rate (for 1 min sample)	1 mm/s
Range	3 m
Differenced Range Rate	$\sqrt{2}$ mm/s
Differenced Range	$3\sqrt{2}$ m

3) MS conventional: two-way range and range-rate from three stations; no simultaneous data.

4) MS simultaneous: two-way range and range-rate from three stations plus simultaneous range and range-rate from pairs of stations during overlapping view periods.

5) MS differenced (or QVLBI): two-way range and range-rate from three stations plus explicitly differenced (two-way minus three-way) simultaneous range and range-rate from pairs of stations during overlapping view periods.

The analytical basis for the information content of MS differenced data is given in References 7-9. It should be emphasized that the latter two strategies use exactly the same data, but in essentially different ways. In the case of QVLBI the explicitly differenced data is weighted conventionally according to the data accuracies given in Table 3, but the two-way data in this case is de-weighted; i. e., it is treated by the filter as if it were less accurate than it really is. The purpose of de-weighting is to minimize the sensitivity to process noise inherent in the two-way data. The two-way data cannot be eliminated completely, as it provides the geocentric range and range-rate information that is largely lost from differencing the simultaneous data. In this study the two-way range and range-rate data for QVLBI were weighted at 1 km and 250 mm/s (for a one minute sample), respectively.

All single-station tracking was assumed to be done from Goldstone (DSS 14). Tracking stations at Madrid (DSS 61) and Johannesburg (DSS 51) were added for the multi-station strategies. These stations were selected because of their favorably overlapping view periods to provide the simultaneous

two-way and three-way data required for QVLBI, as illustrated in Fig. 1. The basic tracking pattern was repeated once each day.

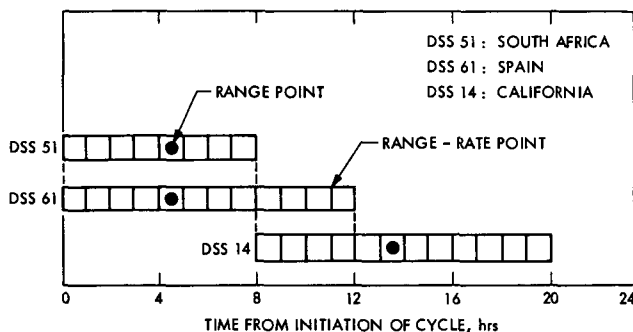


Fig. 1 Simulated Multi-Station Tracking Pattern

#### IV. Baseline Performance

The baseline orbit determination results are given in Fig. 2. The RSS position error at the end of the data arc (E-5d) is shown for each of the five tracking strategies for three cases. The first case is the "optimal" sequential filter (optimal in the sense that the stochastic thrust errors are correctly modeled by the filter's process noise model). This set of bars represents the limiting orbit determination capability under the assumptions of this study with the process noise as represented by the baseline model. There is about an order of magnitude improvement with each change of tracking strategy as one proceeds from SS Doppler-only to SS conventional, MS conventional, and MS simultaneous data, respectively. Since the process noise model is correct, implicit differencing of the simultaneous data by the filter yields good results, and nothing is gained by providing the filter with explicitly differenced data. Thus, the last two strategies give about the same performance in this case.

To fully appreciate the power of QVLBI, one must look at something other than optimal filter performance. That is the main purpose of the second set of bars in Fig. 2. These represent the error in the state estimate resulting from a batch filter solution in the presence of SEP stochastic accelerations; i. e., these are the errors that would result if the real-world stochastic accelerations were actually as represented by our baseline model, but the filter attempted to estimate all thrust variations (including the time-varying components) as if they were biases. (This is equivalent to using infinite correlation times in the process noise model.) It is not intended to suggest that one would ever attempt orbit determination for an SEP mission using a batch filter, but this is a simple means of demonstrating the relative sensitivity of the various tracking strategies to incorrectly modeled (or unmodeled) stochastic accelerations. We have assumed a very bad model, namely, a simple bias. In this case QVLBI (MS differenced data) is clearly superior, giving errors nearly two orders of magnitude smaller than any of the other tracking strategies (but still severely degraded from the optimal filter performance).

The MS simultaneous data in this case offers no advantage over conventional tracking.

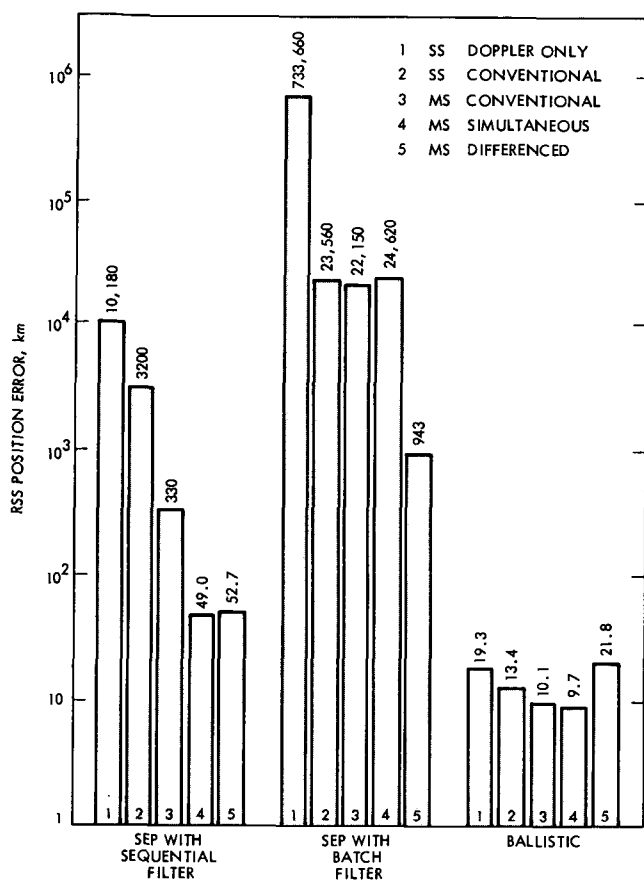


Fig. 2 Baseline Orbit Determination Performance

As a basis for comparison, the ballistic orbit determination performance is also included in Fig. 2. These are the errors that would result if the Encke mission could be flown by a ballistic spacecraft following the SEP trajectory. We assume that the stochastic accelerations are negligible in this case so that data noise and station location uncertainties are the primary error sources. The orbit determination errors in this case are quite small even by ballistic standards due to the favorable geometry of the trajectory near Encke encounter. (The spacecraft is about 0.3 A.U. from Earth with a declination of 60 degrees.) Also, there is little basis for choosing one tracking strategy over another in this case. Even SS Doppler-only tracking gives good performance. The presence of process noise severely degrades the filter's ability to infer position information from range-rate data. This accounts for the dramatic disparity between the ballistic and SEP cases for Doppler-only tracking. Only in the case of optimal sequential filtering with MS simultaneous or MS differenced data are orbit determination errors for SEP within an order of magnitude of those for ballistic spacecraft.

## V. Sensitivity of Optimal Performance to Process Noise Parameters

In this section we consider the effect of changes in the process noise standard deviations and correlation times, but always under the assumption that the filter's model is a correct representation of the actual stochastic environment, thus the optimal orbit determination capability is determined as a function of process noise parameters. For this part of the investigation three tracking strategies were considered: SS conventional, MS simultaneous, and MS differenced. The correlation times were held fixed at baseline values as the standard deviations were varied, and conversely.

First, consider the sensitivity to standard deviations as given in Fig. 3. The standard deviations here are expressed as a ratio with respect to baseline values; i.e., the three process noise standard deviations (two angles and relative thrust magnitude) were always varied by the same multiplicative factor. As expected, the simultaneous and differenced data give essentially the same performance. Of course, the optimal performance of differenced data is always slightly inferior to that of simultaneous data because of the de-weighting of two-way data in connection with the former. Note, however, that as the process noise level increases, the separation of the two curves decreases.

The optimal performance for SS conventional tracking is not only orders of magnitude inferior to the MS strategies, but is also much more sensitive to the process noise level; i.e., the slope of the curve is much greater for SS conventional data. Since the curve in this case is nearly a straight line on the log plot, it can be fit fairly well with an empirical formula of the form

$$\sigma_x = k (\sigma/\sigma_b)^\eta \quad (5)$$

where

$\sigma_x$  = RSS position error, km

$\sigma/\sigma_b$  = ratio of process noise standard deviations to baseline values

$k$  = 3000 for SS conventional data

$\eta$  = 0.89 for SS conventional data

The value of the exponent  $\eta$  being near unity indicates that orbit determination errors are almost linear with the process noise level. The empirical formula (5) does not fit the QVLBI case as well, but would be applicable in the neighborhood of the baseline values with  $k = 55$  and  $\eta = 0.22$ .

The sensitivity to correlation times is given in Fig. 4. For this part of the investigation the three process noise correlation times were assumed to be the same, but a sufficiently wide range of variation was considered to include the baseline values of both the orientation angles (0.125 day) and thrust magnitude (5 days). Once again, SS conventional tracking exhibits the greatest sensitivity. The performance of MS simultaneous and MS differenced data is relatively flat with respect to correlation

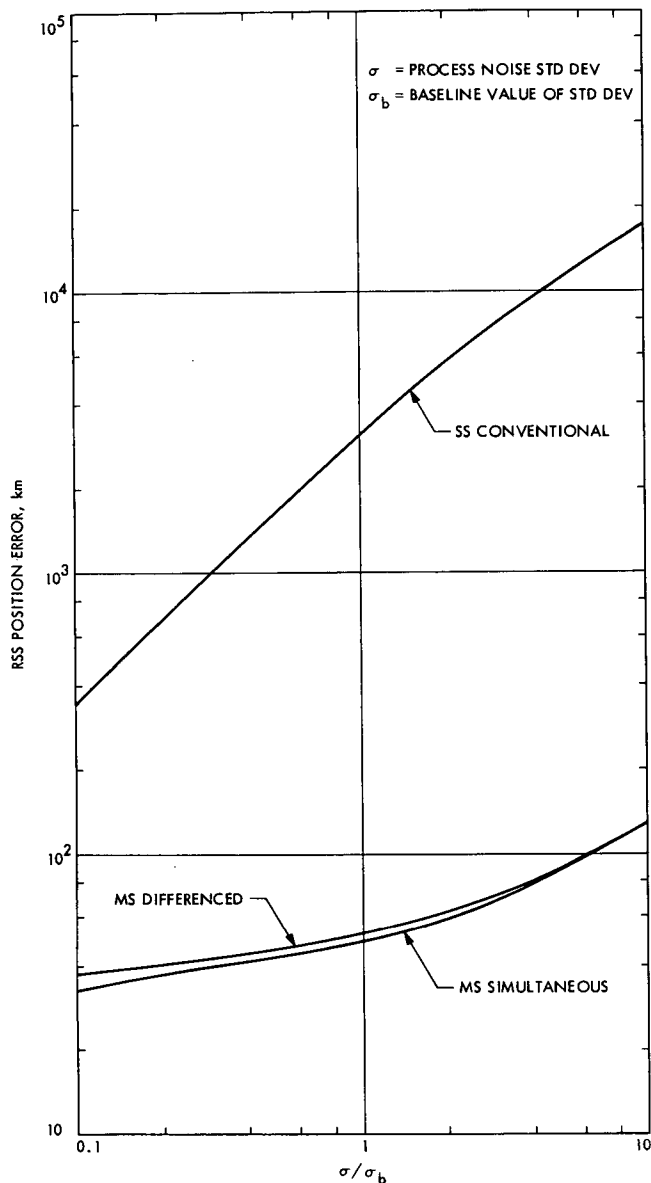


Fig. 3 Sensitivity of Optimal Orbit Determination Performance to Process Noise Standard Deviation

time with the maximum error occurring near one day. Conventional tracking, on the other hand, gives a relatively sharp peak at 0.2 day (about 5 hours) and rapid improvement for larger correlation times (approaching a bias).

#### VI. Sensitivity to Mismodeled Standard Deviation

In this section and the next we consider the more realistic situation where the actual stochastic environment differs from the assumed process noise model of the filter. This kind of analysis not only gives a more realistic assessment of orbit determination capability, but also leads to guidelines for conservative modeling.

Figures 5 and 6 give the orbit determination errors for conventional data and QVLBI (MS differenced data), respectively, for a wide range of actual and assumed standard deviations. The

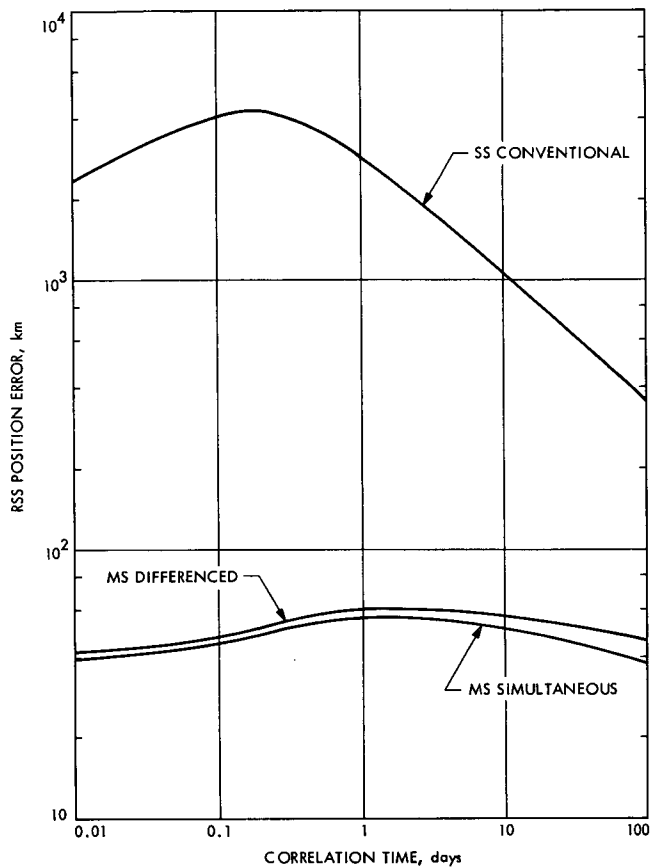


Fig. 4 Sensitivity of Optimal Orbit Determination Performance to Process Noise Correlation Time

dashed line in each of these figures is the locus of minima of the family of curves and is identical with the corresponding optimal performance curve from Fig. 3. In the case of conventional data (Fig. 5) the dashed optimal performance curve divides the suboptimal performance into two regimes. To the left the curves are relatively flat, indicating that using a standard deviation that is too low in the filter's model does not degrade performance significantly from the optimal. Using too large a standard deviation, on the other hand, can degrade performance severely, as indicated by the rapid rise of curves to the right. This pattern does not hold for QVLBI, however. In this case (Fig. 6), the suboptimal performance curves tend to be more nearly symmetric about their minima.

The suboptimal performance of MS simultaneous data with respect to mismodeled standard deviation was found to be virtually indistinguishable from that of MS differenced data as given in Fig. 6, and so those results are not repeated here. At one extreme, with  $\sigma_a/\sigma_b = 4$  and  $\sigma_f/\sigma_b = 0.1$ , the orbit determination errors were 6 percent larger with simultaneous data. At the other extreme, with  $\sigma_a/\sigma_b = 0.25$  and  $\sigma_f/\sigma_b = 10$ , the errors were 5 percent smaller. Thus, if the assumed standard deviation is more than an order of magnitude too low, explicit differencing may offer a very slight advantage.

The suboptimal orbit determination performance can be better understood by analyzing the individual

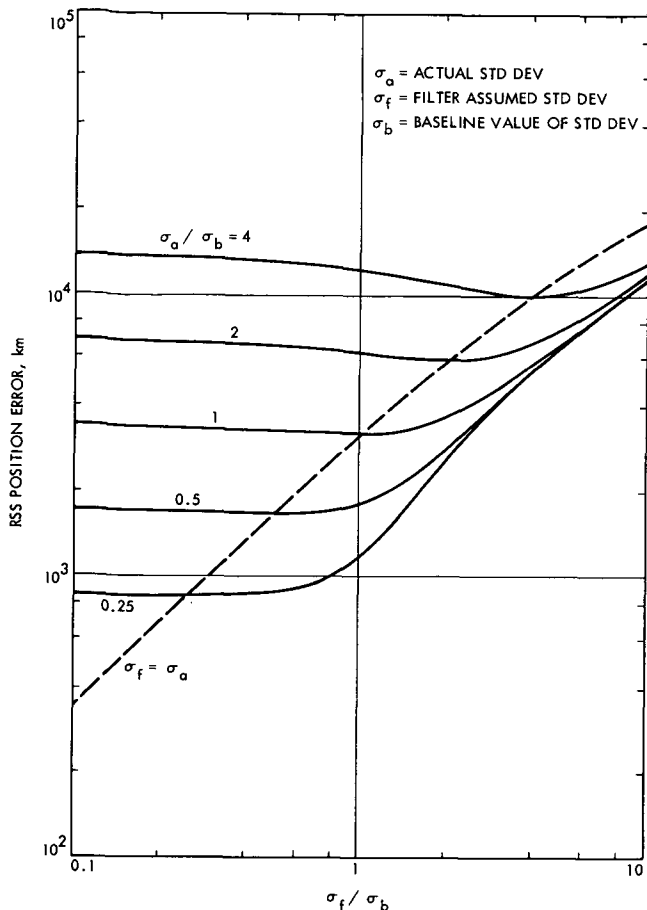


Fig. 5 Sensitivity of SS Conventional Data to Mismodeled Standard Deviation

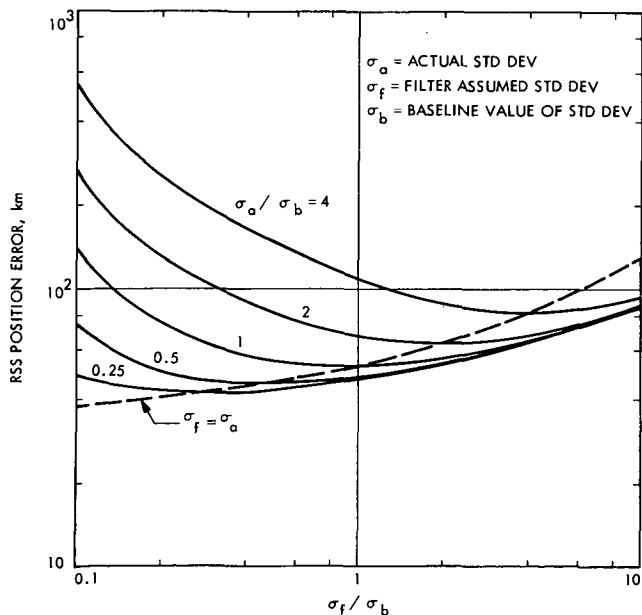


Fig. 6 Sensitivity of MS Differenced Data to Mismodeled Standard Deviation

error sources. This is done in Fig. 7 for conventional data and in Fig. 8 for QVLBI. Fig. 7 shows that with conventional data orbit determination errors are dominated by the effect of process

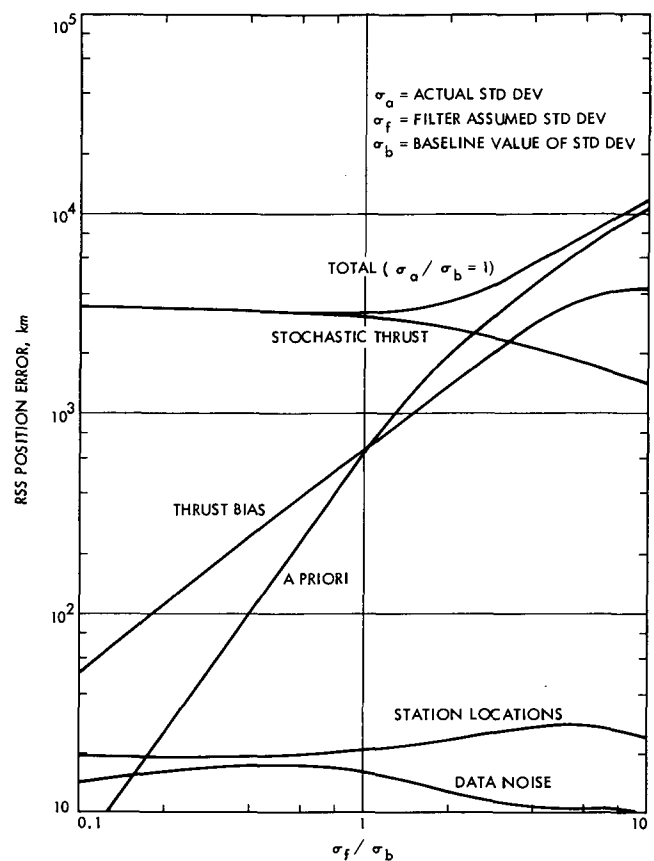


Fig. 7 Contribution of Individual Error Sources to Total Orbit Determination Error for SS Conventional Data

noise. Station locations and data noise are negligible error sources in comparison. As the assumed standard deviation is increased, the filter does a better job of estimating state deviations due to the stochastic thrust components, but the error due to the bias components increases rapidly, with consequent overall degradation. Note that the a priori state uncertainties are still significant even after 15 days of tracking. Indeed, if the assumed standard deviation is much too large, the a priori uncertainties are dominant, indicating that the filter is getting very little information from the data.

In contrast, from Fig. 8 QVLBI is found to be data noise limited, in general. Process noise becomes the dominant error source only when the assumed standard deviation is much too small. Station locations are a significant, but never dominant, error source, and their effect is relatively insensitive to the process noise model. The effect of a priori state uncertainties after 15 days of tracking is too small to appear in Fig. 8.

## VII. Sensitivity to Mismodeled Correlation Time

The preceding section dealt with the case where the dynamic structure of the process noise model (as reflected by the correlation time) was correct, but the level of process noise (as reflected by the standard deviation) was possibly incorrect. In this section we consider the opposite circumstance. This is perhaps the most realistic form of mismodeling, as we usually know more

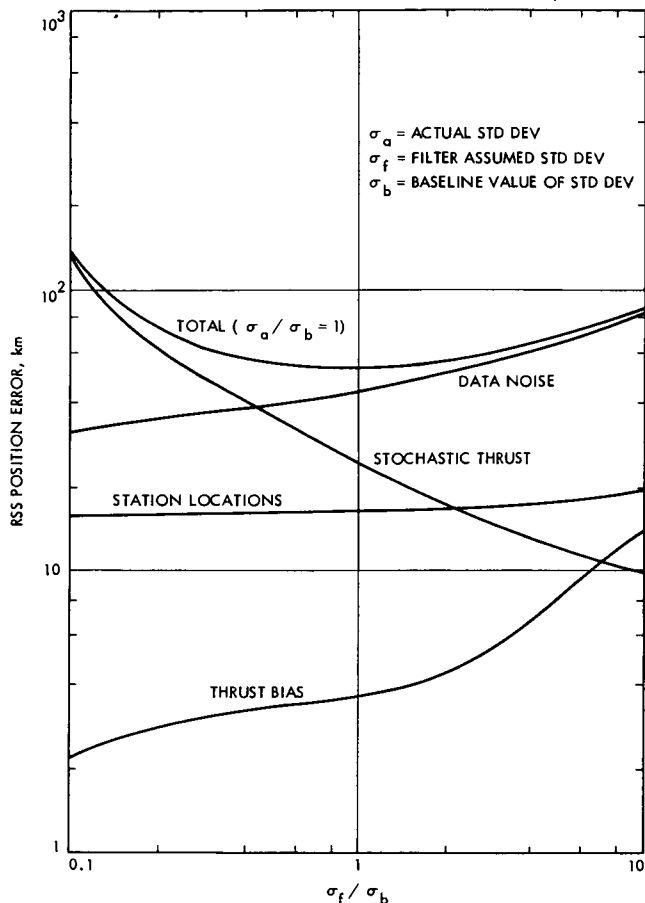


Fig. 8 Contribution of Individual Error Sources to Total Orbit Determination Error for MS Differenced Data (QVLBI)

about the magnitude of stochastic forces than about their underlying dynamics. The results are displayed in Figs. 9-11. Orbit determination errors here are plotted against actual correlation time for various assumed values (cf. Figs. 5 and 6). The primary objective, of course, is to see if any assumed value gives satisfactory performance over a wide range of actual values. As before, to simplify the analysis in this part of the investigation, we have disregarded the difference between orientation angle and thrust magnitude correlation times in the baseline thrust model and have assumed all three components of the process noise to have the same correlation time, both in the filter model and in the supposed actual environment.

The results for conventional data are given in Fig. 9. If the lower envelope for this family of curves were constructed, it would be precisely the optimal performance as a function of correlation time given earlier in Fig. 4. The spread of curves above the lower envelope indicates the extent of potential deterioration of performance from the optimal, assuming an incorrect correlation time. The curves are very widely spaced to the right, indicating considerable loss of performance if a short correlation time is assumed when the actual is large. The spread is much smaller to the left; however, the optimal performance is much worse here; therefore, less deterioration can be tolerated. If the actual correlation time is completely unknown, a conservative choice for the filter model appears to be 0.2 day, as this choice makes the orbit

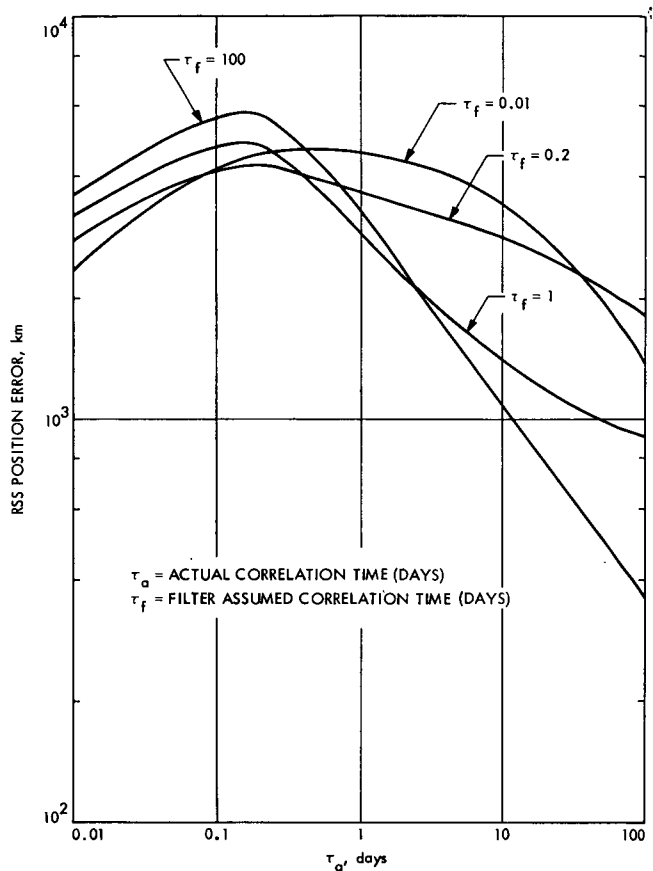


Fig. 9. Sensitivity of SS Conventional Data to Mismodeled Correlation Time

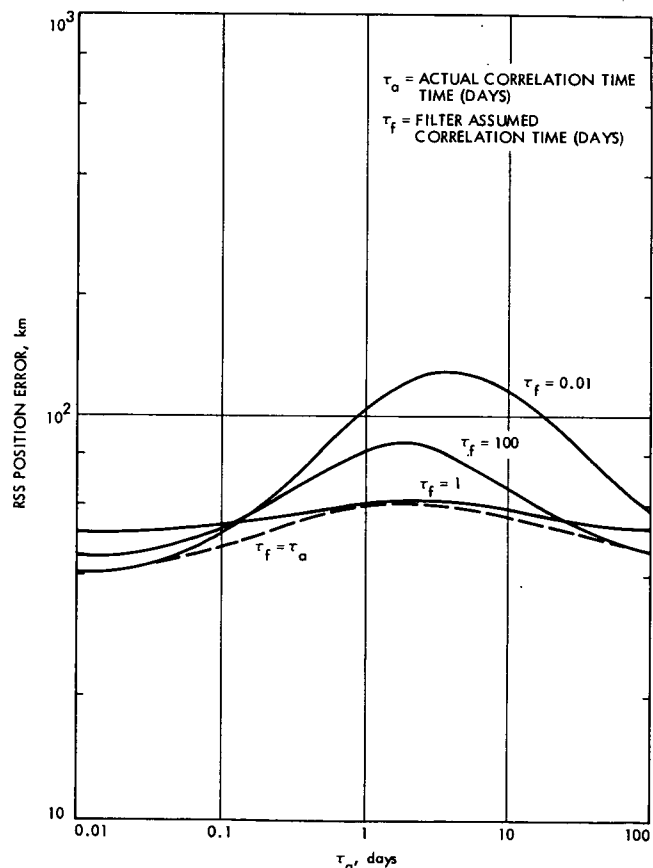


Fig. 10. Sensitivity of MS Differenced Data to Mismodeled Correlation Time

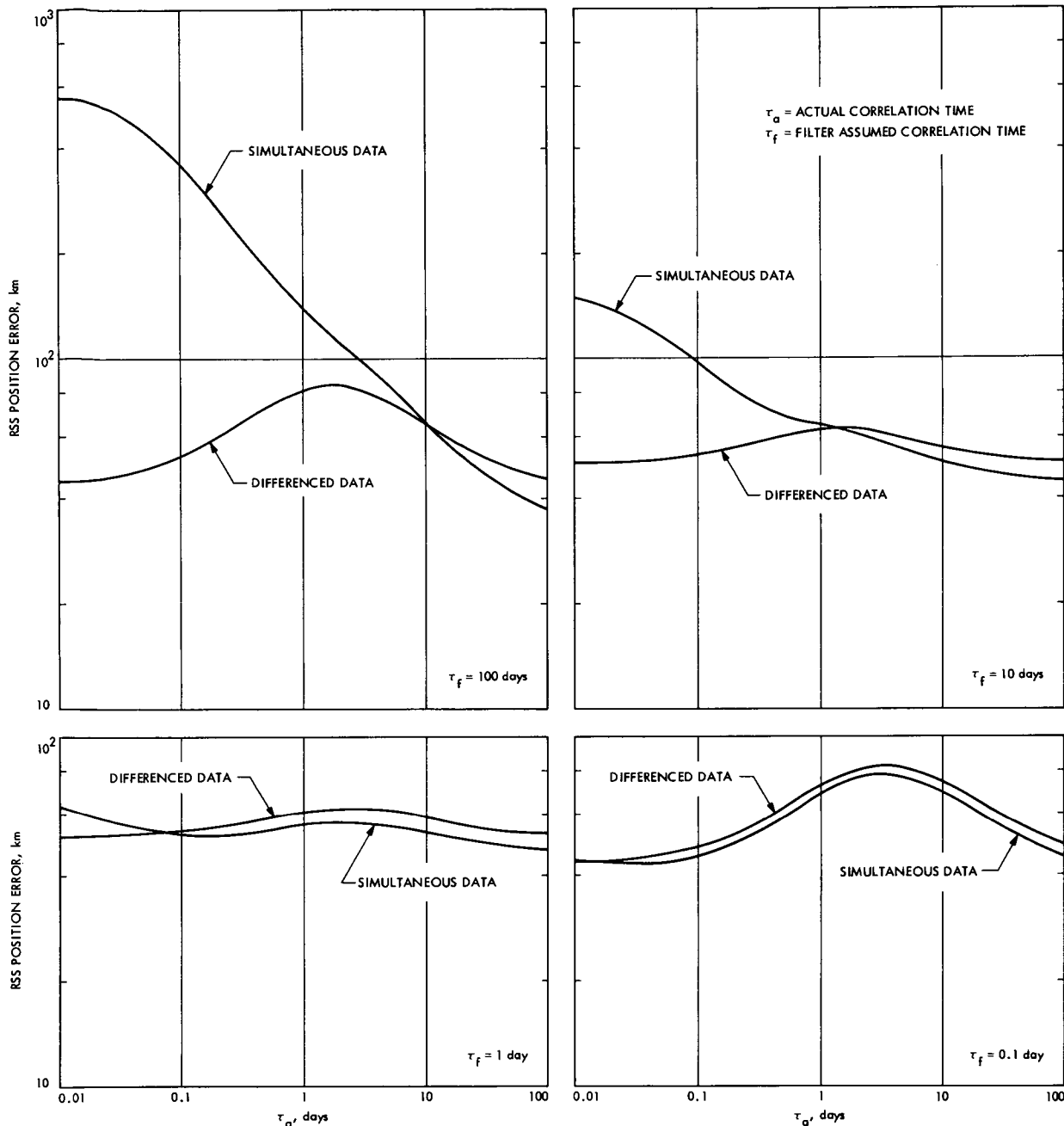


Fig. 11. Comparison of MS Simultaneous and MS Differenced Data with Mismodeled Correlation Time

determination process least sensitive to the actual correlation time. Furthermore, the peak error with  $\tau_f = 0.2$  occurs at  $\tau_a = 0.2$ . This means that if pre-mission analysis is done by computing optimal performance with  $\tau_f = 0.2$ , the actual performance can be no worse regardless of the actual correlation time.

The analogous family of curves for QVLBI is given in Fig. 10, where the lower envelope indicating optimal performance is shown as a dashed line. A correlation time of one day appears to be an excellent choice for the filter model in this case. The curve corresponding to  $\tau_f = 1$  day is remarkably flat and very close to the optimal performance curve over the entire range of actual

correlation times. One question remains: How does simultaneous data compare with explicitly differenced data when the correlation time is mismodeled? The answer is given in Fig. 11 where the performance of these two strategies is compared for four different values of the assumed correlation time. The explicitly differenced data is found to be superior only when the actual correlation time is at least an order of magnitude smaller than the filter's assumed value. For a very large assumed correlation time (e.g., 100 days), the improvement with explicitly differenced data may be considerable (as much as an order of magnitude), but the smaller the assumed correlation time, the less difference there is between the two strategies. For  $\tau_f = 1$  day they give comparable performance.

### VIII. Concluding Remarks

New multi-station tracking techniques have been shown to be superior to conventional single-station tracking for SEP missions, reducing orbit determination errors by orders of magnitude and making SEP navigation capability competitive with ballistic. Explicitly differenced multi-station data is least sensitive to modeling errors and may give satisfactory performance even when the process noise is grossly mismodeled. However, if a reasonably good process noise model is available, or if a conservative model is chosen (i. e., a relatively short correlation time is assumed), explicit differencing of the two-way and three-way data is not required. Although much larger errors result from single-station conventional data, this tracking mode undoubtedly will be used during the cruise portion of an extended SEP mission, where errors are less critical, to minimize the cost of operating the DSN. Therefore, the guidelines developed herein for effective use of conventional data may also be of more than academic interest.

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# APPENDIX

## The Filter Algorithm

The filter algorithm assumes a linear dynamic system of the form

$$\begin{bmatrix} x \\ y \end{bmatrix}_k = \begin{bmatrix} U & V \\ 0 & M \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}_{k-1} + w_k \quad (A-1)$$

with scalar observations

$$z_k = [A \quad B] \begin{bmatrix} x \\ y \end{bmatrix}_k + v_k \quad (A-2)$$

where  $x_k$  is the vector to be estimated,  $y_k$  is the vector of uncertain parameters and stochastic variables to be "considered," but not explicitly estimated, and  $w_k$  and  $v_k$  are white noise processes such that

$$\begin{aligned} E[w_k] &= E[v_k] = 0 \\ E[w_k w_j^T] &= Q \delta_{kj}, \quad E[v_k v_j^T] = R \delta_{kj} \end{aligned} \quad (A-3)$$

where the dependence of  $U$ ,  $V$ ,  $M$ ,  $A$ ,  $B$ ,  $Q$ , and  $R$  upon the index variable  $k$  has been suppressed to simplify the notation. Let  $\hat{x}_k$  be the estimate of  $x_k$  and define the error covariance

$$P(k) = E[e_k e_k^T] = \begin{bmatrix} P_x(k) & P_{xy}(k) \\ P_{xy}^T(k) & P_y(k) \end{bmatrix} \quad (A-4)$$

where

$$e_k = \begin{bmatrix} x - \hat{x} \\ y \end{bmatrix}_k$$

Given  $\hat{x}_0$  and  $P(0)$ , the recursions for propagating the estimate and covariance are

$$P^i(k) = \Phi P(k-1) \Phi^T + Q \quad (A-5)$$

$$K = [P_x^i(k) \quad P_{xy}^i(k)] H^T [A P_x^i(k) A^T + R]^{-1} \quad (A-6)$$

$$P(k) = [I - GH] P^i(k) [I - GH]^T + GRG^T \quad (A-7)$$

$$\hat{x}_k = U \hat{x}_{k-1} + K [z_k - AU \hat{x}_{k-1}] \quad (A-8)$$

where

$$\Phi = \begin{bmatrix} U & V \\ 0 & M \end{bmatrix} \quad H = [A \quad B] \quad G = \begin{bmatrix} K \\ 0 \end{bmatrix} \quad (A-9)$$

Equations (A-5) - (A-8) are intended to show the essential characteristics of the algorithm as simply as possible. Of course, more efficient means of implementation are used in practice. This algorithm is essentially the Schmidt-Kalman filter (12,13) with two minor differences. First, the dynamic model of Eq. (A-1) is slightly more general, allowing the considered variables to be dynamic or even stochastic rather than simple biases. Second, the term  $A P_x^i A^T$  in Eq. (A-6) is replaced by  $H P^i H^T$  in the Schmidt-Kalman filter. This difference is probably not significant in practice, but Eq. (A-6) represents the more conservative choice, as the denominator of the filter gain is smaller in this case, giving a larger value of  $K$  and consequently a little more protection against filter divergence.

Now suppose the covariances  $P(0)$ ,  $Q$ , and  $R$  input to the filter do not represent the true statistics of the ensemble of random variables involved, but rather the true statistics are given by  $\tilde{P}(0)$ ,  $\tilde{Q}$ , and  $\tilde{R}$ , respectively. Then the propagation of the true covariance  $\tilde{P}(k)$  will be given by

$$\tilde{P}^i(k) = \Phi \tilde{P}^i(k-1) \Phi^T + \tilde{Q} \quad (A-10)$$

$$\tilde{P}(k) = [I - GH] \tilde{P}^i(k) [I - GH]^T + G \tilde{R} G^T \quad (A-11)$$

where the filter gain  $K$  (and therefore  $G$ ) is as computed from Eq. (A-6). Thus, the addition of Eqs. (A-10) and (A-11) to the algorithm allows parallel propagation of covariances for the "filter world" and the "real world," as required for this study.

It may not be apparent that the simple scheme described above allows for evaluation of mis-modeled dynamics as well as mismodeled statistics. Suppose, for example, that the effect of mismodeled correlation time for the random component of thrust magnitude is to be evaluated. Then the thrust magnitude dynamics would be included redundantly in the linear system model (i.e., the  $V$  and  $M$  matrices) using both correlation times. The a priori and driving white noise variances corresponding to one correlation time would be set to zero in  $P(0)$  and  $Q$ , and those corresponding to the other correlation time would be set to zero in  $\tilde{P}(0)$  and  $\tilde{Q}$ . As a result, the filter gain is based on one dynamic model, and the propagation of the true covariance takes place with a different dynamic model. Thus, the dynamic modeling error is appropriately accounted for. This is the technique used for the present study.